

Perception-based estimation, classification and clustering for BRDF models of goniochromism and gloss

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11. Abstract <p>Characterizing the appearance of real-world surfaces is a fundamental problem in multidimensional reflectometry, computer vision and computer graphics. For many applications, appearance is sufficiently well characterized by the bidirectional reflectance distribution function. BRDF is one of the fundamental concepts in such diverse fields as multidimensional reflectometry, computer graphics and computer vision. In this paper we study BRDF models of materials that possess complex visual properties such as gloss and goniochromism.</p> <p>We discuss common points and differences between BRDF analysis of glossy and goniochromatic materials in computer graphics and computer vision on one hand, and in metrology and reflectometry on the other hand. We review empirical BRDF models for glossy materials and for goniochromatic materials, and outline possible alternative approaches.</p> <p>We propose a novel, perception-based approach to parameter learning, classification and clustering for BRDF models of goniochromism and gloss. We show that the five basic types of gloss correlate with certain types of shape properties of BRDF manifolds, and perform psychophysical experiments related to development and application of novel machine learning methods to classical BRDF models of computer graphics.</p>		
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1 Introduction

Characterizing the appearance of real-world surfaces is a fundamental problem in multidimensional reflectometry, computer vision and computer graphics [26]. For many applications, appearance is sufficiently well characterized by the BRDF (bidirectional reflectance distribution function).

In the case of a fixed wavelength, BRDF describes reflected light as a four-dimensional function of incoming and outgoing light directions. In a special case of rotational symmetry, isotropic BRDFs are used. Isotropic BRDFs are functions of only three angles. On the other hand, for modelling or describing complicated visual effects such as goniochromism or irradiance, an extra dimension accounting for the wave length has to be added. The BRDF is applied under the assumption that all light falls at a single surface point. The classical device for measuring BRDFs is the gonio-reflectometer, which is composed of a photometer and light source that are moved relative to a surface sample under computer control.

In computer graphics and computer vision, usually either physically inspired analytic reflectance models, or parametric reflectance models chosen via qualitative criteria, are taken for granted and used to model BRDFs. These BRDF models are only crude approximations of reflectance of real materials. Moreover, analytic reflectance models are limited to describing only special subclasses of materials.

In multidimensional reflectometry, an alternative approach is usually taken. One directly measures values of the BRDF for different combinations of the incoming and outgoing angles and then fits the measured data to a selected analytic model using optimization techniques. There are several shortcomings to this approach as well.

A possible alternative to parametric models is in using properly designed simulation studies together with modern data-driven nonparametric estimates of multidimensional manifolds to construct more realistic BRDFs. As an example of this approach, [36] and [37] modelled reflectance of materials in nature as a linear combination of a small set of basis functions derived from analyzing a large number of densely sampled BRDFs of different materials.

There were numerous efforts to use modern machine learning techniques to construct data-driven BRDF models as well. [5] proposed a method to generate new analytical BRDFs using a heuristic distance-based search procedure called Genetic Programming. In [6], an active learning algorithm using discrete perceptual data was developed and applied to learning parameters of BRDF models such as the Ashikhmin - Shirley model [2]. See [25] for an approach based on a combination of statistical and machine learning techniques.

In computer graphics, it is important that BRDF models should be processed in real-time. Computer-modelled materials have to remind real materials qualitatively, but quantitative accuracy is not as important. The picture in reflectometry and metrology is almost the opposite: there is typically no need in real-time processing of BRDFs, but quantitative accuracy is the paramount. In view of this, some of the breakthrough results from computer vision and animation would not fit applications in reflectometry and in many industries.

Another difference with virtual reality models is that in computer graphics measurement uncertainties are essentially never present. This is not the case in metrology, reflectometry and in any real-world based industry (see, e.g., [29], [27] for practical examples). Since measurement errors can greatly influence shape and properties of BRDF manifolds, there is a clear need to develop new methods for handling BRDFs with measurement uncertainties. Suitable statistical and machine learning methods for BRDF data analysis were proposed in [25]. Our novel unified approach aiming at applications requiring both computer graphics representations, as well as physically and perceptually consistent representations of appearance of physical goods, was laid out in [26].

Main results of the present paper concern with perception-based estimation, classification and clustering for BRDF models of glossy reflection. We show that the 5 basic types of gloss correlate with certain types of shape properties of BRDF manifolds, and perform psychophysical experiments related to application of novel machine learning methods to classical BRDF models of computer graphics. This constitutes our novel, perception-based approach to parameter learning, classification and clustering for BRDF models of goniochromism and

gloss.

2 Main definition

The bidirectional reflectance distribution function (BRDF), $f_r(\omega_i, \omega_r)$ is a four-dimensional function that defines how light is reflected at an opaque surface. The function takes a negative incoming light direction, ω_i , and outgoing direction, ω_r , both defined with respect to the surface normal \mathbf{n} , and returns the ratio of reflected radiance exiting along ω_r to the irradiance incident on the surface from direction ω_i . Each direction ω is itself parametrized by azimuth angle ϕ and zenith angle θ , therefore the BRDF as a whole is 4-dimensional. The BRDF has units sr^{-1} , with steradians (sr) being a unit of solid angle.

The BRDF was first defined by Nicodemus in [38]. The defining equation is:

$$f_r(\omega_i, \omega_r) = \frac{d L_r(\omega_r)}{d E_i(\omega_i)} = \frac{d L_r(\omega_r)}{L_i(\omega_i) \cos \theta_i d \omega_i} \quad (1)$$

where L is radiance, or power per unit solid-angle-in-the-direction-of-a-ray per unit projected-area-perpendicular-to-the-ray, E is irradiance, or power per unit surface area, and θ_i is the angle between ω_i and the surface normal, \mathbf{n} . The index i indicates incident light, whereas the index r indicates reflected light.

In the basic definition it is assumed that the wavelength λ is fixed and is the same for both the incoming and the reflected light. In order to model complicated visual effects such as iridescence, luminescence and structural coloration, or to model materials such as pearls, crystals or minerals, as well as to analyze the related data, it is necessary to have an extended, wavelength-dependent definition of BRDFs. Fortunately, formally this new definition is relatively straightforward and is obtained by rewriting equation (1) for $f_r(\lambda_i, \omega_i, \lambda_r, \omega_r)$, where λ_i and λ_r are the wavelengths of the incoming and the reflected light respectively.

3 Glossy materials

BRDF models for glossy materials are usually formed as combined models by summing up the diffuse part and the specular part of the BRDF. The diffuse and the specular part are assumed to be independent within this framework. We treated the diffuse part in details in the Deliverables [32], [33] and in [30], [25].

In this paper, we focus on the BRDF as a whole. We do not treat appearance standards for gloss or goniochromatic colors, as we do not derive models for values of gloss or models for color coordinates. These important characteristics can be derived for each particular explicitly defined BRDF model [31]. On the other hand, for computer graphic reflection models such as the Phong model, the Ward model, or the Cook-Torrance model, there is a correspondence between the parameters of these models and the appearance measurement scale for gloss. This implies that appearance scales, such as gloss, can be used to reparametrize at least some of computer graphics reflection models. An advantage of this reparametrization would be that the value of the parameters of the model can be interpreted in terms of perception or physical attributes. See [53].

We refer to [39] and [34] for more details and references regarding scaling of the gloss and perception of the gloss.

3.1 Phong model

Phong reflectance model [41] is a phenomenological model intended to represent plastic-like specularity. This reflection model describes the way a surface reflects light as a combination of the diffuse reflection of rough surfaces with the specular reflection of shiny surfaces. It is based on an idea that shiny surfaces have small intense specular highlights, while dull surfaces have large highlights that fall off more gradually. The model

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has been extensively used in computer graphics, but nowadays it is considered too crude and replaced by newer approaches. For applications in reflectometry and metrology, this model does not seem to have good fit to real materials.

3.2 Blinn-Phong model

Blinn-Phong model reduces computational overhead of the Phong model via allowing for certain quantities to be interpolated [4]. There is a computational advantage over the Phong model, but the quality of fit is not improved.

3.3 Lafortune model

Lafortune model [23] is a generalization of the Phong model with multiple possible specular lobes.

3.4 Torrance-Sparrow model

Torrance-Sparrow model is a general model representing surfaces as distributions of perfectly specular microfacets [50]. Recently, a more sophisticated model was proposed by [46]. This new model includes as special cases both Lambertian model and the Oren–Nayar model [40], as well as the Torrance–Sparrow model with specular microfacets (see also [47]).

3.5 Cook-Torrance model

Cook-Torrance model [9] is a further advancement of the specular–microfacet Torrance-Sparrow model, accounting for wavelength and thus color shifting.

3.6 HTSG model

The He–Torrance–Sillion–Greenberg model [17] is physically based. This model attempts to take into account a variety of possible physical phenomena such as polarization, diffraction, interference, conductivity, grazing rays.

3.7 Ward model

Ward model [52] is a specular–microfacet model with an elliptical-Gaussian distribution function dependent on surface tangent orientation (in addition to surface normal). It is an approximate model that does not reproduce the Fresnel effect.

3.8 Ashikhmin–Shirley model and distribution–based BRDFs

Ashikhmin–Shirley model [2] allows for anisotropic reflectance, along with a diffuse substrate under a specular surface. A related model is developed in [1]. These models use, in particular, the microfacets idea combined with the Phong reflectance model. Both models are computationally intensive, but are known to give good fit to BRDFs of some real materials.

4 Goniochromatic materials

Goniochromism (also at times called *iridescence*) is the property of certain surfaces that appear to change colour as the angle of view or the angle of illumination changes. The definition of the BRDF in this case has to be modified in order to include the incoming and the outgoing wave lengths. While the modification of the definition is straightforward, the two added dimensions complicate numerical analysis of the problem and make visualization tasks more difficult. Measurement procedures also become more complicated, see [3].

In [15], the authors proposed an approach for parametrization of BRDFs with the wavelength being the added extra dimension. A simplified colored RGB-version of the BRDF is proposed in [16].

Some of the most important causes of goniochromism are listed below. We do not discuss polarization in this article (see [15] for a possible approach).

4.1 Modified Lambertian model for color studies

Lambertian model [24] represents reflection of perfectly diffuse surfaces by a constant BRDF. Because of its simplicity, Lambertian model is extensively used as one of the building blocks for models in computer graphics. Most of the recent studies of colors and light reflection by means of advanced machine learning methods still rely on the Lambertian model. Examples include color studies [49], [43], analytic inference [51], perception studies [14], and face detection [35]. Suitable statistical and machine learning methods for diffuse BRDF data analysis were proposed also in [25].

It was believed for a long time that the so-called standard diffuse reflection materials exhibit Lambertian reflectance, but recent studies with actual BRDF measurements convincingly reject this hypothesis [19], [42], [12], [30], [25], [26].

4.2 Thin-film interference

The *thin-film interference*, i.e. when multiple reflections from two or more semi-transparent surfaces in which phase shift and interference of the reflections modulates the incidental light (by amplifying or attenuating some frequencies more than others). See [18] and [16] for rendering approaches related to thin films.

4.3 Diffraction

Iridescence can also be created by *diffraction*. In the case of diffraction, the entire rainbow of colours will typically be observed as the viewing angle changes. The case of diffraction has been treated in [48].

4.4 Pearlescent coatings

Pearlescent or *nacreous coatings* or pigments possess optical effects that not only serve decorative purposes (such as cosmetics, printed products, industrial coatings, or automotive paints), but also provide important functional roles, such as security printing or optical filters. See [11] for a sophisticated approximation to BRDFs of pearlescent materials.

4.5 Structural coloration

Structural coloration (in both fixed or variable structures): in biological (and biomimetic) uses, colours produced other than with pigments or dyes are called structural coloration. Microstructures, often multilayered, are used to produce bright but sometimes non-iridescent colours: quite elaborate arrangements are needed to avoid reflecting different colours in different directions.

4.6 General purpose models

Even though the Cook-Torrance model [9] and the He-Torrance-Sillion-Greenberg model [17] are rather elaborated physically based models, accounting for color shifting, and even for some complicated effects such as polarization, diffraction, interference, conductivity and grazing rays in the case of the HTSG model, it has been noticed [16] that these optical effects are not represented accurately by these BRDFs. See the diffraction of the light on a CD-disk example.

There seems to be no universally applicable systematic approach to constructing BRDFs for any of these cases. BRDF models are mostly processed on a case-to-case basis, even though there are some impressive examples of realistic illumination for specific types of surfaces. Books [10] and [8] contain a good overview of the current state-of-the art in this field of computer graphics and list a number of interesting special cases.

5 Perception-based analysis of BRDF models

In this section, we focus on perception-based estimation, classification and clustering for BRDF models of glossy reflection. Perception-based analysis is possible for goniochromatic effects as well, but it has to rely on another type of perception studies related to colors (see D. B. Judd's classical work [22]).

A classic attempt to classify glossy appearances based on human perception was performed in [20] (see also [22] and references therein). These authors established a framework that dominates research on gloss perception to the present day. Their gloss dimensions are used as the basis of many industrial metrics for gloss measurement and specification, see [13].

In short, Hunter suggests to classify glossy appearances into at least those 6 different kinds of gloss: (1) specular gloss, identified by shininess; (2) sheen, identified by surface shininess at grazing angles; (3) contrast gloss, corresponding to contrasts between specularly reflecting areas and other areas of surfaces; (4) absence-of-bloom gloss, corresponding to the absence of reflection haze or smear adjacent to reflected high lights; (5) distinctness-of-reflected-image gloss, corresponding to the distinctness of images reflected in surfaces; (6) absence-of-surface-texture gloss, characterized by the lack of surface texture and surface blemishes.

This classification of gloss is not expected to be final and complete, as there are several types of glossiness that are not explained by this analysis. Moreover, viewed under different conditions, surfaces present more than six different types of glossiness [20]. It is also worth noticing that there still seems to be not enough data for comprehensive studies of perception of different types of glossy materials.

As was shown by Hunter and Judd in a series of works, see [22], the (1)-(5) types of gloss correlate (but not coincide) with certain types of aggregated light reflection statistics. These aggregated statistics are, correspondingly, the following ones: (1) specular gloss at the incoming angle 60 degrees; (2) specular gloss at grazing incoming light angles; (3) ratio of specular gloss at 45 degrees to nonspecular reflectance factor; (4) ratio of somewhat off-specular reflectance factor to specular gloss; (5) ratio of very slightly off-specular reflectance factor to specular gloss. Of course, part (2) here is obvious, while the other parts represent some quite nontrivial physical observations.

Since the gloss type (6) corresponds to BTDFs and SVBRDFs rather than to BRDFs, we leave this case for future work.

In modern mathematical language [26], we would say that these 5 types of gloss correlate with certain types of shape properties of BRDF manifolds. Hunter and Judd's hypotheses imply even a stronger type of statements: namely, shape properties of certain sub-manifolds or even properties of simple layers of BRDFs can serve as summary statistics for building perception-based classifiers of gloss. We explore this idea in the following Section.

5.1 Perception-based estimation for BRDF models of gloss

For modelling purposes, we use the freely available BRDF Explorer by Walt Disney Research (<http://www.disneyanimation.com>) see also [7]. BRDF Explorer is very instrumental in our supervision of parameter learning procedures, as this program allows to pick BRDF model parameters with an arbitrary precision, and it produces an illuminated image of an object made of the material with this particular BRDF.

We trained our classifiers and learning procedures via illuminating a sphere in the BRDF Explorer and then asking a group of people to do grading of different appearance properties based on the pictures they saw on the screen. Parameter learning is achieved via the classic simple bisection procedure, but can be also modified to use more sophisticated approaches. It would be interesting to expand our experiment to include transfer learning framework [21].

*Due to Open Access policies associated with publication of the present technical report, we omit description of the underlying psychophysical experiment and do not provide any raw data.

5.1.1 Phong model

We choose the model variation with division by $N \cdot L$, and the angle option of double θ in the BRDF Explorer. For simplicity, we always take $\theta_i = 60$ and $\phi_i = 0$. There remains one model parameter n .

Experiment suggests that $n = 1$, $1 < n < 4$, $4 \leq n < 8.9$, $8.9 \leq n < 20$ and $n \geq 20$ all correspond to different types of gloss.

5.2 Blinn-Phong model

For the Blinn-Phong model with parameters defined as for the basic Phong model, we observe that $1 \leq n < 3$, $3 \leq n < 39$, and $39 \leq n$ correspond to different types of gloss possible within this model.

5.3 Cook-Torrance model

For the Cook-Torrance model, we use the double θ option, include F and G in the construction, and do not multiply by $N \cdot L$. We remain with the almost-2-dimensional parameter space of (m, f_0) , with $m \geq 0$ and $0 \leq f_0 \leq 1$.

The line segment $m = 0$, $0 \leq f_0 \leq 1$ gives a class of unlit surfaces. The curvilinear trapezoid between the points $(0, 0)$, $(0, 1)$, $(0, 173; 0)$ and $(0, 143; 1)$ gives another type of gloss, with one shiny white ball on the illuminated sphere. In principle, two other types of gloss can be seen if one allows only for $0 \leq m \leq 0.3$. If one allows for arbitrary nonnegative m and f_0 , we get at least two more types of pictures, but some of these might be physically impossible for real objects.

5.4 Perception-based classification for BRDF models of gloss

The previous Subsection contains, together with parameter estimates, an outline of how a supervised classifier for BRDF models can be trained. We obviously have here a multiclass classification procedure. In general, supervised classification has been extensively treated in statistics and machine learning. However, in the present scenario, the exact number of classes that would be encountered for each particular BRDF model, is not known in advance. Moreover, this number is clearly different for different BRDF models. All we know from Judd and Hunter's works [22], [20], is that we can expect 6 classes of gloss for real life objects, but it is not known how many types of gloss a complicated real or artificial BRDF can possibly produce.

This type of classification problems is consistently solved by recently developed I-SVMs, or Support Vector Machines with infinite number of classes [28].

5.5 Perception-based clustering for BRDF models of gloss

It is possible to build perception-based clustering algorithms for spaces of BRDF parameters. One of the ways here is to use some kind of a perception-based distance between images. See [45], [44] for examples of such metrics. For each point in the parameter space, we can produce a sphere illuminated with the corresponding BRDF. Running a distance-based clustering algorithm on the set of these illuminated spheres, induces a set of clusters on the original space of BRDF's parameters.

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