

Sensitivity analysis for BRDF measurements of standard reference materials

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11. Abstract <p>Characterizing the appearance of real-world surfaces is a fundamental problem in multidimensional reflectometry, computer vision and computer graphics. For many applications, appearance is sufficiently well characterized by the bidirectional reflectance distribution function. BRDF is one of the fundamental concepts in such diverse fields as multidimensional reflectometry, computer graphics and computer vision. In this paper, we treat BRDF measurements as samples of points from high-dimensional non-linear non-convex manifold. We argue that statistical analysis of BRDF measurements has to account both for nonlinear structure of the data as well as for ill-behaved noise. Standard statistical methods can not be safely directly applied to BRDF data. In this paper we construct a new class of measures of quality of fit for BRDF models. We will use these measures to evaluate influence of measurement and model errors on both real and simulated BRDFs. Our measure of distance between BRDFs is leading to physically meaningful conclusions for basic applications of BRDF data analysis. This is an important property as our analysis has to be applicable to physical objects and real data.</p>		
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1 Introduction

Characterizing the appearance of real-world surfaces is a fundamental problem in multidimensional reflectometry, computer vision and computer graphics. For many applications, appearance is sufficiently well characterized by the BRDF (bidirectional reflectance distribution function) [10].

In the case of a fixed wavelength, BRDF describes reflected light as a four-dimensional function of incoming and outgoing light directions. In a special case of rotational symmetry, isotropic BRDFs are used. Isotropic BRDFs are functions of only three angles. The BRDF is applied under the assumption that all light falls at a single surface point. The classical device for measuring BRDFs is the gonio-reflectometer [2], which is composed of a photometer and light source that are moved relative to a surface sample under computer control.

In computer graphics and computer vision, usually either physically inspired analytic reflectance models, or parametric reflectance models chosen via qualitative criteria, are taken for granted and used to model BRDFs. These BRDF models are only crude approximations of reflectance of real materials. Moreover, analytic reflectance models are limited to describing only special subclasses of materials.

In multidimensional reflectometry, an alternative approach is usually taken [3], [2]. One directly measures values of the BRDF for different combinations of the incoming and outgoing angles, and then it becomes possible to fit the measured data to a selected analytic model using optimization techniques. There are several shortcomings to this approach as well.

In computer graphics, it is important that BRDF models should be processed in real-time. Computer-modelled materials have to remind real materials qualitatively, but quantitative accuracy is not as important. The picture in reflectometry and metrology is almost the opposite: there is typically no need in real-time processing of BRDFs, but quantitative accuracy is the paramount. In view of this, some of the breakthrough results from computer vision and animation would not fit applications in reflectometry and in many industries.

Another difference with virtual reality models is that in computer graphics measurement uncertainties are essentially never present. This is not the case in metrology, reflectometry and in any real-world based industry. Since measurement errors can greatly influence shape and properties of BRDF manifolds, there is a clear need to develop new methods for handling BRDFs with measurement uncertainties.

In this paper, we treat BRDF measurements as samples of points from a high-dimensional and highly non-linear non-convex manifold. We argue that any realistic statistical analysis of BRDF measurements has to account both for nonlinear structure of the data as well as for a very ill-behaved noise. Standard statistical methods for analysis of errors can not be safely directly applied to BRDF data.

In computer science literature on BRDFs, there is a few papers that study quality of fit of BRDF models to real data, see [13], [18]. Even fewer studies attempt to do a more rigorous statistical analysis, either for quality of fit or for influence of errors. Most of these studies use the (most standard) L_2 -norm, see [1] [17]. Overall, very few papers explore BRDFs with statistical tools beyond standard textbook methods (see [11] for an example of this type of research).

In this paper we propose and construct a new class of measures of quality of fit for BRDF models. We will use these measures to evaluate influence of measurement and model errors on both real and simulated BRDFs. Our measure of distance between BRDFs is leading to physically meaningful conclusions at least for some basic applications of BRDF data analysis. This is a useful property as our analysis has to be applicable to physical objects and real data.

2 BRDF definition

The bidirectional reflectance distribution function (BRDF), $f_r(\omega_i, \omega_r)$ is a four-dimensional function that defines how light is reflected at an opaque surface. The function takes a negative incoming light direction, ω_i ,

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and outgoing direction, ω_r , both defined with respect to the surface normal \mathbf{n} , and returns the ratio of reflected radiance exiting along ω_r to the irradiance incident on the surface from direction ω_i . Each direction ω is itself parameterized by azimuth angle ϕ and zenith angle θ , therefore the BRDF as a whole is 4-dimensional. The BRDF has units sr^{-1} , with steradians (sr) being a unit of solid angle.

The BRDF was first defined by Nicodemus in [14]. The definition is:

$$f_r(\omega_i, \omega_r) = \frac{d L_r(\omega_r)}{d E_i(\omega_i)} = \frac{d L_r(\omega_r)}{L_i(\omega_i) \cos \theta_i d \omega_i} . \quad (1)$$

where L is radiance, or power per unit solid-angle-in-the-direction-of-a-ray per unit projected-area-perpendicular-to-the-ray, E is irradiance, or power per unit surface area, and θ_i is the angle between ω_i and the surface normal, \mathbf{n} . The index i indicates incident light, whereas the index r indicates reflected light.

3 Basic properties of distributions in BRDF data

In our choice of data analysis procedures for BRDF models, we have to take into account specific properties of BRDF data. It is important to notice that, due to the complicated structure of measurement devices, outliers are possible in the data. Additionally, due to technical difficulties in measuring peak values of BRDFs (see [16], [15]), we have to count on the fact that certain (even though small) parts of the data contain observations with big errors. This also leads us to conclusion that, even for simplest additive error models, we cannot blindly assume that random errors are identically distributed throughout the whole manifold. Additionally, missing data are possible and even inevitable for certain angles. Measurement angles themselves can be also arbitrary and non-uniformly distributed.

In view of the above arguments, a useful data analysis procedure for any BRDF model has to exhibit certain robustness against outliers [5] and dependent or mixed errors [7].

A statistical procedure has to be universal enough in the sense that it has to be applicable to BRDF samples without requiring extra regularity in the data set, such as uniformly distributed (or other pre-specified) design points, pre-specified large number of measurements, or absence of missing values. This observation suggests that simple and robust methods are more practical for BRDF data than complicated (even if possibly asymptotically optimal) methods, as the later class of procedures has to rely on rather strict regularity assumptions about the underlying model [6].

BRDF measurements represent a sample of points from a high-dimensional and highly non-linear non-convex manifold. Moreover, these measurements are collected via a nontrivial process, possibly involving random or systematic measurement errors of digital or geometric nature. These two observations suggest that any realistic statistical analysis of BRDF measurements has to account both for nonlinear structure of the data as well as for a very ill-behaved noise. Standard statistical methods typically assume nice situations like i.i.d. normal errors, and can not be safely directly applied to BRDF data. Our study of parameters for Lambertian models in [11] clarified certain pitfalls in analysis of BRDF data, and helped to understand and develop more refined estimates for more realistic BRDF models.

4 Statistical analysis of BRDF models

Suppose we have measurements of a BRDF available for the *set of incoming angles*

$$\Omega_{inc} = \{\omega_i^{(p)}\}_{p=1}^{P_{inc}} = \{(\theta_i^{(p)}, \varphi_i^{(p)})\}_{p=1}^{P_{inc}} . \quad (2)$$

Here $P_{inc} \geq 1$ is the total number of incoming angles where the measurements were taken. Say that for an incoming angle $\{\omega_i^{(p)}\}$ we have measurements available for angles from the *set of reflection angles*

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$$\Omega_{refl} = \bigcup_{p=1}^{P_{inc}} \Omega_{refl}(p), \quad (3)$$

where

$$\Omega_{refl}(p) = \left\{ \omega_r^{(q)} \right\}_{q=1}^{P_{refl}(p)} = \left\{ (\theta_r^{(q)}, \varphi_r^{(q)}) \right\}_{q=1}^{P_{refl}(p)}, \quad (4)$$

where $\{P_{refl}(p)\}_{p=1}^{P_{inc}}$ are (possibly different) numbers of measurements taken for corresponding incoming angles.

Overall, we have the set of random observations

$$\left\{ f(\theta_i^{(p)}, \varphi_i^{(p)}, \theta_r^{(q)}, \varphi_r^{(q)}) \mid (\theta_i^{(p)}, \varphi_i^{(p)}) \in \Omega_{inc}, (\theta_r^{(q)}, \varphi_r^{(q)}) \in \Omega_{refl}(p) \right\}. \quad (5)$$

Our aim is to infer properties of the BRDF function (1) from the set of observations (5). In general, the connection between the true BRDF and its measurements is described via a stochastic transformation T , i.e.

$$f(\omega_i, \omega_r) = T(f_r(\omega_i, \omega_r)), \quad (6)$$

where

$$T : \mathcal{M} \times \mathcal{P} \times \mathcal{F}_4 \rightarrow F_4, \quad (7)$$

with $\mathcal{M} = (M, \mathfrak{A}, \mu)$ is an (unknown) measurable space, $\mathcal{P} = (\Pi, \mathfrak{P}, \mathbb{P})$ is an unknown probability space, \mathcal{F}_4 is the space of all Helmholtz-invariant energy preserving 4-dimensional BRDFs, and F_4 is the set of all functions of 4 arguments on the 3-dimensional unit sphere S^3 in \mathbb{R}^4 .

Equations (6) and (7) mean that there could be errors of both stochastic or non-stochastic origin. In this setting, the problem of inferring the BRDF can be seen as a statistical inverse problem. However, contrary to much literature on this subject, we do not assume linearity of the transformation T , we do not assume that T is purely stochastic, and we do not assume an additive model with zero-mean parametric errors, as these assumptions do not seem realistic for BRDF measurements.

Of course, this setup is intractable in full generality, but for special cases we would be able to obtain quite general solutions. Related (weak) regularity conditions were discussed in [8].

5 Foundations of sensitivity analysis of BRDF models

How do we evaluate influence of measurement errors? How do we measure quality of fit of BRDF models? We need a "measure of distance" between BRDFs. This measure must be reasonable from a qualitative viewpoint and it has to be physically reasonable, instead of merely providing us a number for a pair of BRDFs.

There are many measures of distance and quasi-distance used in mathematics and statistics: L_p , $1 \leq p < +\infty$, L_∞ , Sobolev distances [19], Kullback-Leibler information divergence [4], Mahalanobis [12] and Minimum Description Length distances, chi-squared distance used in correspondence analysis [5], and other types of metrics and pseudo-metrics. Are they appropriate for applications related to BRDFs?

In computer science literature on BRDFs, there is a few papers that study quality of fit of BRDF models to real data. Even fewer studies attempt to do a more rigorous statistical analysis, either for quality of fit or for influence of errors. Most of these studies use the (most standard) L_2 -norm. Namely, for BRDFs $f_1 = f_1(\omega_i, \omega_r)$, $f_2 = f_2(\omega_i, \omega_r)$

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$$L_2^2(f_1, f_2) = \int \int \int \int_{\Omega} \|(f_1 - f_2)(\theta_i, \phi_i, \theta_r, \phi_r)\|^2 d\theta_i d\phi_i d\theta_r d\phi_r, \quad (8)$$

where Ω is the set containing all possible values of parameters, in this case,

$$\Omega = \Theta_i \times \Phi_i \times \Theta_r \times \Phi_r. \quad (9)$$

Notice that Ω can be different for other BxDFs or in those cases where other restrictions are imposed.

It is known from both mathematics and statistics that not all distances are equally appropriate in all applications. More specifically, for a problem related to a functional class \mathcal{F} , only those distance are potentially applicable that are either defined or could be extended for all functions from \mathcal{F} .

As an example, for problems dealing with extremely heavy-tailed distributions with infinite variance, both the L_2 -distance as well as related least squares methods, are not applicable. This example explains why we should not blindly use L_2 -metrics for analysis of BRDF data. Instead, we develop an appropriate "metric" for BRDFs.

Definition Let \mathcal{F}_4 be the *space of all Helmholtz-invariant energy preserving 4-dimensional BRDFs*. More precisely,

$$\mathcal{F}_4 = \{f : \Theta_i \times \Phi_i \times \Theta_r \times \Phi_r \rightarrow \mathbb{R}_+ \text{ such that} \\ f(\theta_i, \phi_i, \theta_r, \phi_r) = f(\theta_r, \phi_r, \theta_i, \phi_i) \text{ for all } \theta_i, \phi_i, \theta_r, \phi_r \text{ and} \quad (10)$$

$$\int_{\Theta_r \times \Phi_r} f(\theta_r, \phi_r, \theta_i, \phi_i) \cos \theta_r d\theta_r d\phi_r \leq 1 \text{ for all } \theta_i, \phi_i\}. \quad (11)$$

Any good metric for BRDFs shall not only be defined over \mathcal{F}_4 , but this metric also should be leading to physically meaningful conclusions. This is necessary as our analysis has to be applicable to physical objects.

Notice that the energy conservation inequality (11) does not even imply

$$\int_{\Omega} \|f\|^2 d\mu_{\Omega} < +\infty,$$

where μ_{Ω} is a product measure on Ω . Therefore, \mathcal{F}_4 contains functions with extremely heavy tails.

There are two main directions for further developments.

Direction 1. Perform studies and do virtual experiments for the case when heavy-tailed distributions are used to model specular peaks.

Pros: 1. Virtual experiments with heavy-tailed distributions are of much interest on their own. Moreover, they help to locate the borderline between "nice and smooth" models and their parameter range, and extreme cases where most of the standard data analysis methods are not applicable. This is an important part of analysis of sensitivity.

2. Approximations are avoided. In particular, if BRDFs with peaks are precisely measured (see planned experiments in [16], [15]), all of the information will be utilized.

Cons. Surprisingly, currently there is no method to generate extremely heavy-tailed distributions with any kind of reasonable accuracy! This is a big obstacle for applications in metrology where a high precision is needed. A paper [9] is aiming to solve this problem by providing an algorithm for generating extreme heavy tails with perfect fit.

Direction 2. Smooth out outliers, gross errors and heavy-tailed peaks, but in a mathematically and physically reasonable way.

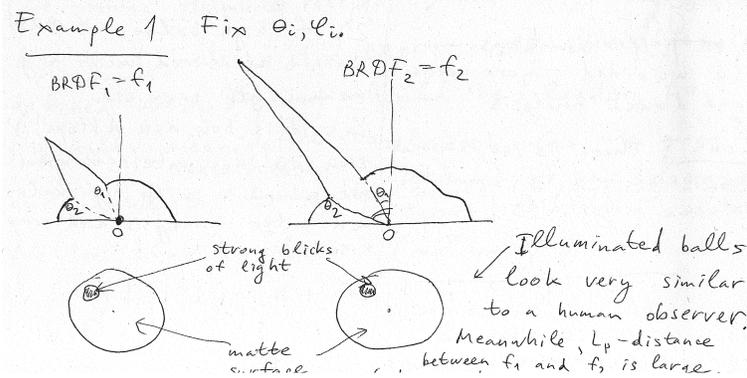


Figure 1: Large L_p -distance, small difference in appearance

Pros: 1. Modeling and data analysis become methodologically easier.

2. As extremely high values of BRDFs at specular peaks are not measured with any reasonable precision, then no grossly inaccurate or badly corrupted measurement with high value could enter analysis and comparisons.

Cons. A choice of metric of distance between BRDFs is important, as smoothing out the extremes has to give a result that is physically sensible.

Direction 1 will be our future direction of research, especially after high precision peak measurements will be available at CNAM. In present conditions, it makes the most sense to follow Direction 2. Now let us focus once again on a proper metric for \mathcal{F}_4 .

We start with series of motivating examples. Notice that majority of materials (and even some diffuse reflection standards) have a specular component, at least for a range of $\theta_i^{spec} \leq \theta < \pi/2$, for some incoming angle $\theta_i^{spec} > 0$. The specular component is responsible for shiny areas on the surface of the illuminated object. They are very distinct for the human observer.

Our main idea can be formulated as follows:

$\left(a \text{ "distance" } d \text{ between BRDFs is physically reasonable} \right)$ if and only if $\left[\left\{ d(f_1, f_2) \text{ is small} \right\} \text{ if and only if } \left\{ \text{two balls illuminated according to } f_1 \text{ and } f_2 \text{ would look similar to a human observer} \right\} \right]$.

Example 1. Fix θ_i, ϕ_i . In the example on Figure 1, illuminated balls look very similar to a human observer. Meanwhile, an L_p -distance between f_1 and f_2 is large. From Example 1 we conclude that position of the BRDF's peak is more important for a human observer than the height and shape of the peak.

Example 2. Here an L_p -distance can be made small, since a very thin second peak of f_2 gives a small contribution to the integral. However, the difference between f_1 and f_2 is substantial for a human observer, as the second shiny place on the right ball is clearly noticeable. From Example 2 we conclude that the number of

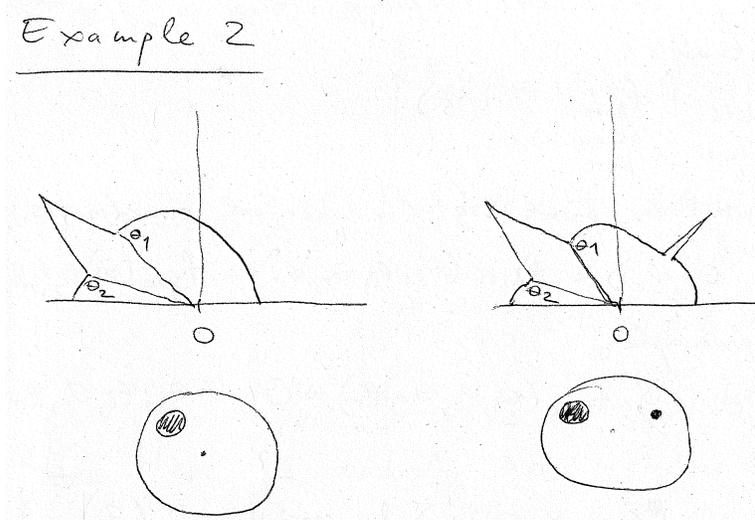


Figure 2: Small L_p -distance, but visible difference in appearance

BRDF's peaks is important. Missing any peaks or adding unnecessary peaks is noticeable, so a good distance on \mathcal{F}_4 should penalize for missing or added peaks.

Example 3. For the same fixed θ_i, ϕ_i , see Figure 3. From Example 3 we conclude that locations of peaks are also important for an observer. This point is well addressed by L_p -distances as well, though.

We had to focus on peaks in our examples, because L_p - and L_∞ -metrics are dominated by peaks.

6 Measure of distance between BRDFs

The following reasonable decomposition is typically used in BRDF studies:

$$BRDF(\theta_i, \varphi_i, \theta_r, \varphi_r) = f_{diff}(\theta_i, \varphi_i, \theta_r, \varphi_r) + f_{spec}(\theta_i, \varphi_i, \theta_r, \varphi_r). \quad (12)$$

The above decomposition is given pointwise and in spherical coordinates. It is assumed that all BRDFs and their decompositions are μ_Ω -measurable functions. We assume that either for all $(\theta_i, \varphi_i, \theta_r, \varphi_r) \in \Omega$

$$f_{diff}(\theta_i, \varphi_i, \theta_r, \varphi_r) \cdot f_{spec}(\theta_i, \varphi_i, \theta_r, \varphi_r) = 0, \quad (13)$$

or, almost equivalently, that

$$\int_{\Omega} |f_{diff} \cdot f_{spec}|(\theta_i, \varphi_i, \theta_r, \varphi_r) d\mu_{\Omega} = 0. \quad (14)$$

Therefore, for each BRDF $f \in \mathcal{F}_4$ we have a decomposition of the parameter space Ω

$$\Omega = \Omega_{diff}(f) \cup \Omega_{spec}(f), \quad \mu_{\Omega}(\Omega_{diff}(f) \cap \Omega_{spec}(f)) = 0. \quad (15)$$

As before, we denote $\omega_i = (\theta_i, \varphi_i)$, $\omega_r = (\theta_r, \varphi_r)$, $\omega = (\omega_i, \omega_r)$. Our goal is to construct μ_{BRDF} , a "metric" on the BRDF-space \mathcal{F}_4 . Then the following decomposition is possible:

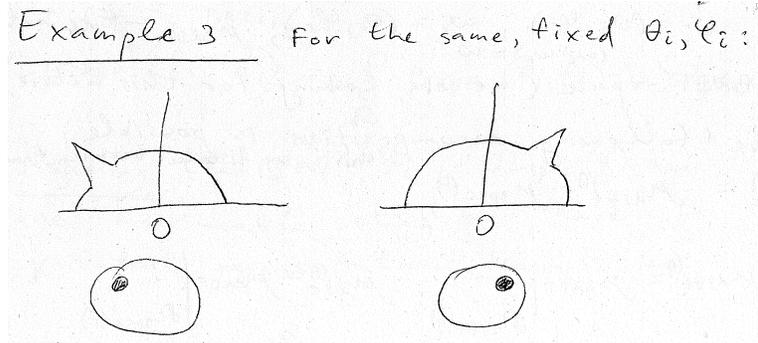


Figure 3: Locations of peaks are important

$$\mu_{BRDF}(f) = \mu_{diff}(f) + \mu_{spec}(f) \quad (16)$$

where we denote $\mu_{BRDF}(f) = \mu_{BRDF}(f, 0)$, a distance between f and 0 (i.e., an absolutely light-absorbing surface). Decomposition (16) holds if we put, say,

$$\mu_{diff}(f) = \mu_{BRDF}(f) \Big|_{\Omega_{diff}(f)}, \quad \mu_{spec}(f) = \mu_{BRDF}(f) \Big|_{\Omega_{spec}(f)}. \quad (17)$$

An important point is that μ_{diff} and μ_{spec} could and should be different.

In this paper we use the following rule to define a "specular" and a "diffuse" part of the BRDF. Notice that our definition of Ω_{spec} and Ω_{diff} are formal and technical, and they do not have to correspond exactly to the physical phenomenae of specular or diffuse reflection. Our definition allows us to split the area of glossy reflection between Ω_{spec} and Ω_{diff} without having to treat glossy reflection separately.

Definition

$$\Omega_{spec}(f) = \{\omega \mid f(\omega) \geq \Delta_1\} \quad (18)$$

$$\Omega_{diff}(f) = \Omega \setminus \Omega_{spec}(f) \quad (19)$$

Definition Let f_1, f_2 be BRDFs. Define the *element of distance* between f_1 and f_2 in \mathcal{F}_4 at a point ω as

$$\rho_{BRDF}(f_1, f_2)(\omega) = \begin{cases} \rho_{diff}(f_1(\omega), f_2(\omega)), & \omega \in \Omega_{diff}(f_1) \cap \Omega_{diff}(f_2); \\ \rho_{spec}(f_1(\omega), f_2(\omega)), & \omega \in \Omega_{spec}(f_1) \cap \Omega_{spec}(f_2) \cap \\ & \{\omega \mid \rho_{spec}(f_1, f_2)(\omega) < \Delta_1\}; \\ \Delta_2, & \omega \in \Omega_{spec}(f_1) \cap \Omega_{spec}(f_2) \cap \\ & \{\omega \mid \rho_{spec}(f_1, f_2)(\omega) \geq \Delta_1\}; \\ \Delta_2, & \omega \in (\Omega_{spec}(f_1) \cap \Omega_{diff}(f_2)) \\ & \cup (\Omega_{diff}(f_1) \cap \Omega_{spec}(f_2)). \end{cases} \quad (20)$$

Here Δ_1 and Δ_2 are pre-selected constants, with Δ_2 penalizing missing or added peaks and smoothing out the influence of large peaks and of gross errors in measurements of peaks. Δ_1 determines the level where we switch the regime in our analysis.

Definition We define the *metric (distance)* μ_{BRDF} on \mathcal{F}_4 as follows. For any BRDFs $f_1, f_2 \in \mathcal{F}_4$, set

$$\mu_{BRDF}(f_1, f_2) = \int_{\Omega} \rho_{BRDF}(f_1, f_2)(\omega) d\mu_{\Omega}(\omega). \quad (21)$$

As μ_{diff} we can reasonably use any L_p -measure. It is possible to use an L_{p_1} -measure μ_{spec} , possibly for some $p_1 \neq p$.

A number of truncated measures has been considered in robust statistics.

7 Basic properties of new distances on the space of BRDFs

7.1 Basic properties

Proposition 7.1 Assuming that for any $f \in \mathcal{F}_4$, the sets $\Omega_{spec}(f)$ and $\Omega_{diff}(f)$ are measurable, we have the following equivalent representation (decomposition):

$$\mu_{BRDF}(f_1, f_2) = \mu_{diff}(f_1, f_2) \Big|_{\Omega_{diff}(f_1) \cap \Omega_{diff}(f_2)} + \quad (22)$$

$$\mu_{spec}(f_1, f_2) \Big|_{\Omega_{spec}(f_1) \cap \Omega_{spec}(f_2) \cap \{\omega \mid \rho_{spec}(f_1, f_2)(\omega) < \Delta_1\}} + \quad (23)$$

$$\Delta_2 \cdot \mu_{\Omega} \left(\Omega_{spec}(f_1) \cap \Omega_{spec}(f_2) \cap \{\omega \mid \rho_{spec}(f_1, f_2)(\omega) \geq \Delta_1\} \right) \quad (24)$$

$$\Delta_2 \cdot \mu_{\Omega} \left((\Omega_{spec}(f_1) \cap \Omega_{diff}(f_2)) \cup (\Omega_{diff}(f_1) \cap \Omega_{spec}(f_2)) \right). \quad (25)$$

Proof Follows directly from (20) and (21). \blacksquare

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Lemma 7.2

$$\mu_{\Omega}(\Omega) = \frac{\pi^2}{8}. \quad (26)$$

Proof $\mu_{\Omega}(\Omega) = \mu_{\Omega}(S^3)$, where S^3 is 1/8 of a 3-sphere in \mathbb{R}^4 . For the octant S^3 ,

$$\mu_{\Omega}(S^3) = \frac{\pi^2}{4} \cdot \frac{1}{\Gamma(4/2)} = \frac{\pi^2}{8}.$$

■

Proposition 7.3 *Under the above assumptions, both sets in (24) and (25) are μ_{Ω} -measurable, and*

$$\begin{aligned} \mu_{\Omega} \left(\Omega_{spec}(f_1) \cap \Omega_{spec}(f_2) \cap \{\omega \mid \rho_{spec}(f_1, f_2)(\omega) \geq \Delta_1\} \right) &< +\infty, \\ \mu_{\Omega} \left((\Omega_{spec}(f_1) \cap \Omega_{diff}(f_2)) \cup (\Omega_{diff}(f_1) \cap \Omega_{spec}(f_2)) \right) &< +\infty. \end{aligned}$$

Proof Follows directly from Lemma 7.2. ■

Note that calling μ_{BRDF} a measure or a distance is an abuse of terminology, since μ_{BRDF} does not always satisfy the axioms for distances. It would be more appropriate to call μ_{BRDF} either quasi- or pseudo-distance. A similar situation is true for the Kullback-Leibler information measure (or information distance) [4]. This is not actually a distance, but its other properties make the information distance one of the most popular distance measures used in statistical model selection or in machine learning in general.

8 Conclusions

BRDF is one of the fundamental concepts in such diverse fields as multidimensional reflectometry, computer graphics and computer vision. Usually, either physically inspired analytic reflectance models, or parametric reflectance models chosen via qualitative criteria, are taken for granted and used to model BRDFs. These BRDF models are only crude approximations of reflectance of real materials. Moreover, analytic reflectance models are limited to describing only special subclasses of materials.

In computer graphics and vision, it is important that BRDF models should be processed in real-time, but quantitative accuracy is not as important. In reflectometry and metrology, it is the opposite: there is typically no need in real-time processing of BRDFs, but quantitative accuracy is the paramount. In view of this, some of the breakthrough results from computer vision and animation would not fit applications in reflectometry and in many industries.

Another difference with virtual reality models is that in computer graphics measurement uncertainties are essentially never present. This is not the case in metrology, reflectometry and in any real-world based industry. Since measurement errors can greatly influence shape and properties of BRDF manifolds, there is a clear need to develop new methods for handling BRDFs with measurement uncertainties.

In this paper, we treated BRDF measurements as samples of points from high-dimensional non-linear non-convex manifold. We have shown that statistical analysis of BRDF measurements has to account both for nonlinear structure of the data as well as for ill-behaved noise. Standard statistical methods can not be safely directly applied to BRDF data. We clarified certain pitfalls in analysis of BRDF data.

We proposed a new class of measures of quality of fit for BRDF models. We will use these measures to evaluate influence of measurement and model errors on both real and simulated BRDFs. Our measure of distance between BRDFs is leading to physically meaningful conclusions for basic applications of BRDF data analysis. This is a useful property as our analysis has to be applicable to physical objects and real data.

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